Chapter 4

Game people play

- The Mathching card game and the Go Fish task are by Anne Mathieu of Irem Clermont-Ferrand[2] and [1]. Since audio was fast, also unusable as standalone, picked a video explaining the rules. Couldn't use the five tips podcast.
- The Monty-Hall problem is taken from Jenny Gage and David Spiegelhalter's Teaching Probability [6].
- The SET game workout is adapted from [4]
- The Project B is inspired by [7]
- The dobble problem is adapted from Activity Handouts by [3] The Activity guide was helpfull too. One can use dobble-like generator here [9] and here [8]. For further reading check [5] and [10].

4.1 Game : Go fish

Vocabulary To play cards, a game of cards, deck (US), suit (Spades ♠, Hearts, ♡ , Diamond ♢, Clubs ♣), rank (Ace, King, Queen, Jack, 10-2), two of Hearts, Jack of Spades, joker, to shuffle, to deal, hand, dealer, to draw, draw pile.

Exercise 1 — 👂 **How to play.** Listen anf fill the blanks https://youtu.be/hRpXLSMdve0 (1'30")

The object of the game is to the most four of a kind. Setup : ____________ a standard __________ of playing cards, and deal 5 cards to each player. Players look at their cards but do not show them to others. Pick a player to go first, then play proceeds ______________.

On your turn you ask for one other player for any card rank[1]. For example : "Bob, do you have any threes?". The asked player must give you all the cards ______________ the rank from their hand, then you get to take another turn. If the asked player doesn't have any of the asked cards in their hand they say: "Go Fish." You then must _________ 1 card from the top of the deck. If you draw the card you had just asked for, show it and take another turn, otherwise end your turn.

Whenever a player gets all four of cards of the same rank, a "4 of a kind"[2], they immediately remove those cards from their hand and place them grouped together face up in front of themselves. Whenever a player runs out of cards in their hand they immediately draw 5 new cards from the deck. When the deck runs out of cards, play continues until every card in every player's __________ is gone.

The game is now over. Each player receives 1 point for each 4 of a kind match they have in front of them. The player with the most points wins.

Exercise 2 — **Variations of Go Fish.** Go Fish is a variant of the traditional *Happy Families* card game played in the UK with a set of 44 picture cards featuring families of four (mother, father, son, daughter), based on occupation types (the Vet, the Postman, the Baker etc.).

What is this game called in France? Explain the rules of the French game.

[1] The player who is "fishing" must have at least one card of the rank that was asked for in their hand.
[2] This is called a book

Exercise 3 — How to win at Go Fish.

1. Imagine you are dealt 5 cards at the start of the game and you have 3 Jacks and 3 Aces, What rank would you ask on your first turn and why?

2. Write down 3 tips you could give to win. Share your ideas with the rest of the class.

Exercise 4 — Probability and cards.

1. A card is selected at random from a pack of 52 cards. Find the probability that the card will be the King of Clubs.

2. What is the probability that the chosen card is either a Queen or a 7?

3. What is the probability that the card will be a face card or a Heart?

4.2 Game : Prime climb

Vocabulary To shuffle, to draw a card, to roll a die, to roll dice.

Exercise 1 — **How to play.** Watch “Prime Climb” Instructional Video `https://youtu.be/usBHrp6s4xY` and play a few games.

Objective Get both your pawns to 101 exactly.

Set-up

1. Lay out the board and shuffle the 24 Prime Cards
2. Choose your color. Place two pawns on 0
3. Use the “Go first” dice to decide who will play first.

Game Play Players take turns. A turn consists of four phases:

1. **ROLL** the dice. The two numnbers your roll will be use *individually* to move your pawns.
   In the case of **DOUBLES**, use the number you rolled four times instead of twice.
   You must use all your rolls each turn except on the turn you win.
2. To **MOVE** your pawn, *ADD, SUBTRACT, MULTIPLY* or *DIVIDE* the number your pawn is on by the number you rolled.
3. **BUMP** If you end your Move phase with either of your pawns on the same space as another pawn (including your own), you MUST send the pawn you landed on to 0.
4. **DRAW** a Prime Card if you end your Move phase with one or more of your pawns on a red prime.
   Keeper cards are kept face up for a future turn. Can't be played the turn you draw it.
   Action cards are played immediately.

Exercise 2 — **investigate.** Answer the following riddles.

1. How can you get two pawns from 0 to 101 in four rolls (that's eight numbers) without any number appearing on a die more than once?
2. It's possible to solve the last problem with the additional stipulation that three of your four rolls sum to the same number. Can you find out how?

3. a) Your pawn is at 100. What is the probability of reaching 101 on your next roll? (You don't have to use both dice rolls when you reach 101, though of course you may.)

		first die												
		1	2	3	4	4	5	5	6	6	7	8	9	10
second die	1													
	2													
	3													
	4													
	5													
	6													
	7													
	8													
	9													
	10													

b) What if your pawn was at 99?

4. In the middle of a certain game, Katherine and I were down to a single pawn each. Hers was on 24, and mine was on a certain unnamed number. I rolled a little too forcefully, and the dice went off the table on her side.

- "Ha," she said. "If you had been at 0, you could have hit me."
- "Then I can hit you from where I am!" I said.

What number was I on?

4.2.1 Solutions for Puzzles

for full article https://archive.nytimes.com/wordplay.blogs.nytimes.com/2014/05/19/primo/

1. Solution 1: Dice 1 $2 \times 6 \times 8 + 5 = 101$ and Dice 2 : $10 \times 9 + 4 + 7$

   Alternate solution, first roll: (10,1) -> P1 at 11, P2 at 0

   Second roll: (9,2) -> P1 at $9 \times 11 + 2 = 101$, P2 at 0

   Third roll: (8 , 3) -> P2 at $(0 + 3) \times 8 = 24$

   Fourth roll (4;5) -> P2 at $24 \times 4 = 96 + 5$.

2. Solution 1: $2 + 12 = 8 + 4 = 5 + 7$ give all 12. So 3 roll out of 4 add up to 12.

   For the alternate solution, 3 rolls all sum to 11.

3. 35% when pawn is at 100, and 34% when pawn is at 99.

4. The dice are either 6 and 4, either 8 and 3. He was on a number that was able to hit.

   As Golden Dragon explained: “Since he could have hit 24 if he had been at 0, that means that the dice must have been either 8 and 3 or 6 and 4. So whatever number he really was at needs to be able to reach 24 with either pair. Trying various combinations, I found 16 to satisfy this. 16 minus 8 = 8 times 3 is 24. 16 times 6 = 96 divided by 4 is 24.”

   Ravi noticed that 1 could have solved the problem as well: “From 0, it is only 0 + 8 X 3 or 0 + 4 X 6 that hit 24. And when on 1, you could use 1 X 8 X 3 or 1 X 4 X 6 to get to 24 with either of the rolls.”

4.3 Game of Nim

The term "Nim" refers to a whole class of games. In general, two players take turns removing stones from one or more rows. Rules vary, resulting in a panoply of interesting games. We present just a few in ways that are particularly suited to a Festival or Math Circle. Nim is a two-person, perfect-knowledge game of strategy. *Perfect knowledge* means that there are no hidden cards or moves, and no dice to roll, and therefore that both players know everything about their posi tions.

By varying the number of piles, and the rules for picking up stones, we get different versions. Nim is often played as a *misère* game, in which the player to take the last stone loses. Nim can also be played as a normal play game, where the player taking the last stone wins. This is called normal play because the last move is a winning move in most games.

Exercise 1 — 👂 How to play.

1. Watch *The unbeatable Game of Nim* https://youtu.be/EiqJcQ7YxHw to learn a version of Nim.
2. a) How many counters is each player allowed to remove ?(make a sentence)

   b) How many counters was there at the beggining of each game ?

   c) What is the strategy to win, that is to leave a single counter at the end ?

   d) How did Josie's dad manage to win even if Josie guessed the right strategy ?

Exercise 2 — Analyze a One-Row Nim game.

A number of stones are lined up. Two players take turns picking up stones. In one turn, a player can pick up 1, 2, 3, 4, or 5 stones. A player cannot pass: she must take at least one stone and cannot take more than five stones. The player to pick up the last stone loses the game.

1. Suppose there are 12 stones at first. One of the players can always win the game. Which one? What should her strategy be?

   You can play several games with your partner first :

Game 1	1	2	3	4	5	6	7	8	9	10	11	12
Game 2	1	2	3	4	5	6	7	8	9	10	11	12
Game 3	1	2	3	4	5	6	7	8	9	10	11	12
Game 4	1	2	3	4	5	6	7	8	9	10	11	12
Game 5	1	2	3	4	5	6	7	8	9	10	11	12
Game 6	1	2	3	4	5	6	7	8	9	10	11	12
Game 7	1	2	3	4	5	6	7	8	9	10	11	12
Game 8	1	2	3	4	5	6	7	8	9	10	11	12
Game 9	1	2	3	4	5	6	7	8	9	10	11	12
Game 10	1	2	3	4	5	6	7	8	9	10	11	12

2. Suppose the game starts with 15 stones, rather than 12. Which player now has a winning strategy? What is it?

3. What if the game starts with 18 stones? 19 stones?

4.4 Game : a Card trick using Monty Hall!

Exercise 1 — Recall the Monty Hall dilemma.

1. Complete the probability tree.

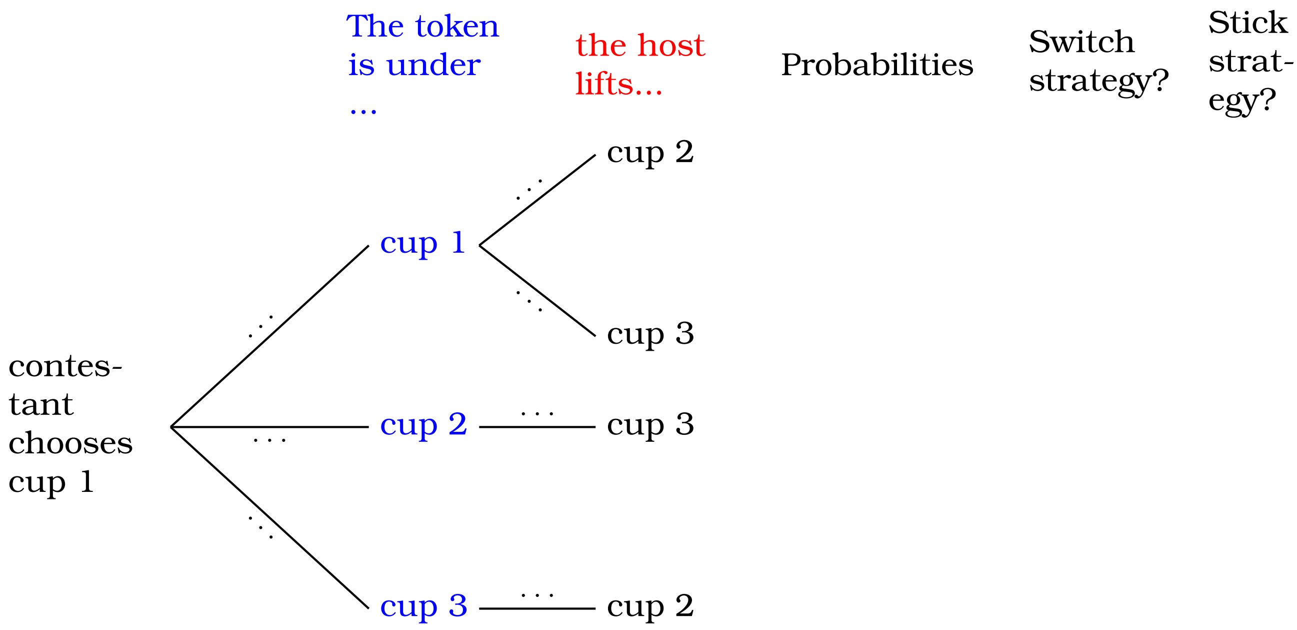


Figure 4.1: Tree diagramm assuming the token is hidden under cup 1.

2. Fill in the blanks and compare the probabilities of winning if you switch or stick.

   If player B always switched, he will only lose if Cup 1 actually covered the sweet. Since this happens with probbility ____, a switching strategy will only lose ____ of the time.

Exercise 2 — a New Monty Hall Variation!.

1. Let's watch `https://youtu.be/v0SOG3tYEbM` together.
2. How many cards are dealt by player A? How many are chosen by player B?

3. What cards are revealed by player A? Did he know where the Ace was?

4. The trick is designed to people who already know the Monty hall problem. Those would assume that switching cards is the better strategy.

   Draw the Tree Diagramm for this card game, and compare the probabilities of winning if you switch or stick.

4.5 Project A: Learning Mathemagic Tricks

Situation You and a partner are to select a magic trick or a puzzle that involves math. You will perform it in front of your classmates and explain the maths behind it.

Possible strategies Watch several videos, and pick a trick that you like or that you think is easier to perform.

Notes

- Ask teacher for material or make a sample of your game.
- Practice makes perfect! Try the trick to be able to perform it with ease.
- Prepare your introduction. You need to capture the audience's attention.
- Use the video explaining the trick to understand its mechanics and some of the math behind it.
- Prepare clear diagrams if necessary to illustrate.

Magic tricks Following are a small selection of some tricks and the math behind them.

- **The Government Strikes Back!** `https://youtu.be/TN9pZWXMgmk`.
  (Pigeonhole principle, number of permutations).
- **A truly random prediction that always makes cents** `https://youtu.be/0FpermM9NaA`
  (Divisibility rule of 9) (alternative `https://youtu.be/-6cUIWjJGtE`)
- the **21-card trick** `https://youtu.be/d7dg7gVDWyg`
  (Arithmetic, logic)
- the **27-card trick** `https://youtu.be/l7lP9y7Bb5g`
  (Arithmetic, base-3 numbers1)
- 🌵 **Same size card trick** `https://youtu.be/V3uNDe_i_1Y`

Project : Games people play.

Date ....

Name(s) ....

Picked theme ....

Teacher assessment

Content & teamwork	**/2**
•Used only english and was fully involved during preparation	/1
•All the elements required are present	/1
The quality of English	**/9**
•Grammar	1 2 3 /3
•Syntax	1 2 3 /3
•Vocabulary	1 2 3 /3
Pronunciation	**/6**
•Stress pattern	1 2 3 /3
•Phonology	1 2 3 /3
Overall quality of the presentation and interaction	**/3**
•Student is addressing the audience and not reading his/her notes.	/1
•The performance of the trick is fluid.	/1
•Student understands and answers the audience's questions.	/1
Total	**/20**

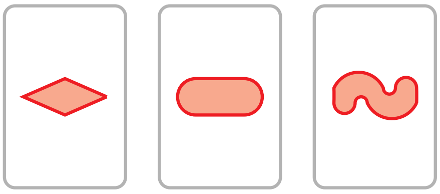
Student Self-assessment To assess your work and what you have learned during this project, answer the following questions.

1. What did I like about this project?
2. What did I not like about it?
3. Was the math hard to understand?
4. Is my work sufficient? Could I have provided more? How?

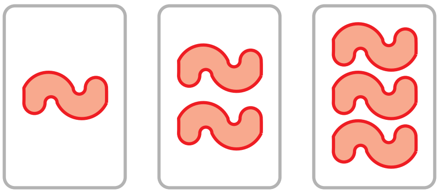
4.6 Game of SET

Set is a multi-player game played with a special deck of cards. Each card is uniquely determined by four features : number, color, shape and shading which can vary as follow:

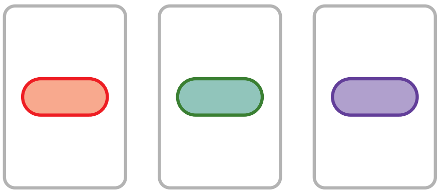
Figure 4.2: All four feetures off a card of SET



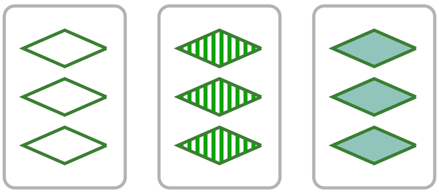
(a) *3 shapes:* diamond, oval and squiggle



(b) *3 numbers:* 1, 2 or 3 shapes per card



(c) *3 colors:* red, green or purple

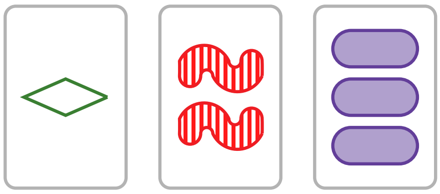


(d) *3 shadings:* plain/empty, striped/hashed or solid/-full

Definition 4.1 — A SET. is a collection of 3 cards such that for each feature, either all the cards in the trio share the same value for that feature, or else no two have the same value for it.

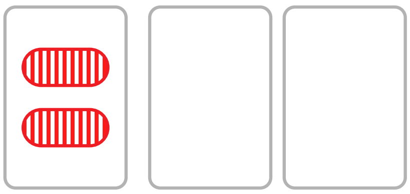
■ **Exemple 4.1**

- "one plain green diamond"
- "two striped red squiggles"
- and "three solid purple ovals" form a set.

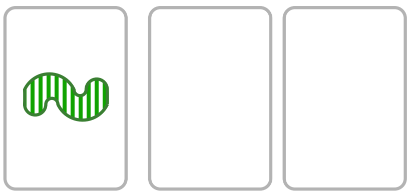


Exercise 1

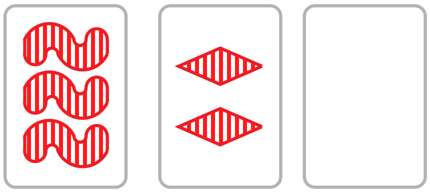
1. Complete to form a SET with the same shape, the same color, the same number of symbols and all different shading.



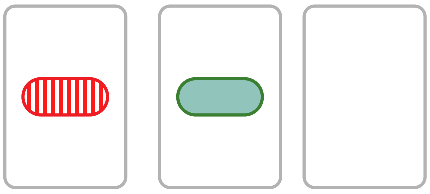
2. Complete to form a SET with different shapes, different colors, and different numbers of symbols and the same shading.



3. Find a card to form a SET. Explain why it is a SET in a simple sentence.



4. Find a card to form a SET. Explain why it is a SET in a simple sentence.



Exercise 2 — How to play.

Listen anf fill the blanks https://youtu.be/NzXDfSFQ1c0 (2'30")

> **Instructions**
> To play, we start with a shuffled deck and the dealer deals 12 cards face up on the table. Your goal, as a player, is to find a SET. Once you find one, you call it out, collect the SET (assuming that it is a legitimate one), and the dealer deals 3 more cards. The object of the game is to collect the most SETs. One point is given for each SET. Highest score wins.
>
> A player must call SET before picking up the cards. There are no turns, first player to call SET has few seconds to pick his cards. If the SET is incorrect, he/she loses one point.

Daily Set challenge https://www.setgame.com/set/puzzle

Vocabulary

1. **to deplete, depleted, depleted**

   The lake was depleted of water. ....
2. **to resume, resumed, resumed** ....
3. **to stripe, striped, striped**

   Zebras have unique striped pattern. ....
4. **to strip, stripped, stripped**

   The tank was stripped down piece by piece ....

   The general was stripped of his rank. ....

Exercise 3 — analyze the game.

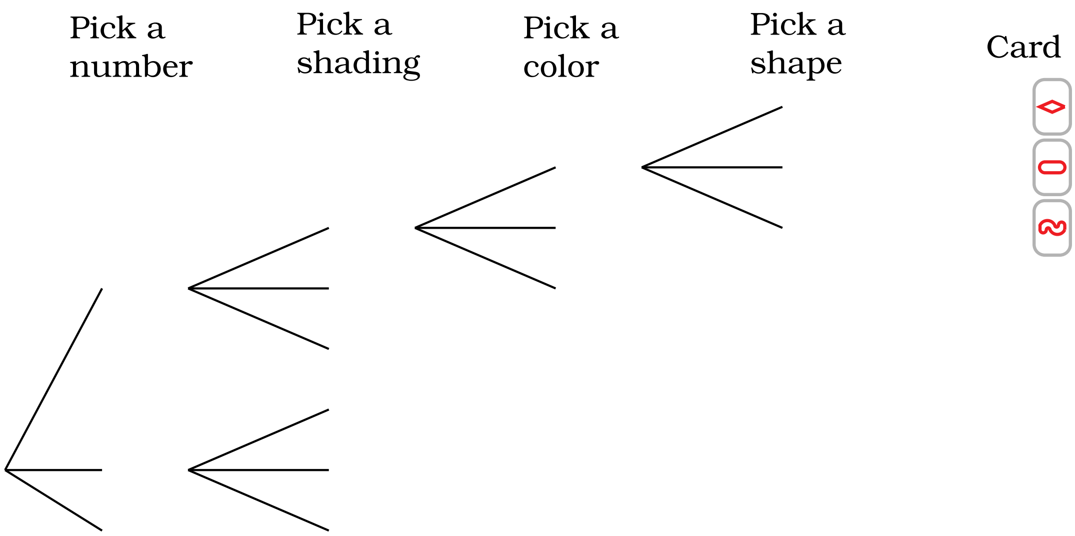
1. How many different cards in a SET deck? *You can use the text below to explain yourself*

To make a card we ....characteristics (to pick/picking/picked).

Each characteristic has ....

We can draw a ....to count all ....

There are a total of ....



2. Show that for every two cards there is a unique third card that will make a set

(you can use the blanks to organise your thoughts)

(different from/same/similar/similarly). Take the characteristic ________________.

Either the first two cards have the same ________________ , and the third card must have ________________. Either the first two cards have ________________, and then the third card must have ________________ and there is ________________ possibility for that. ________________ for the other three characteristics.

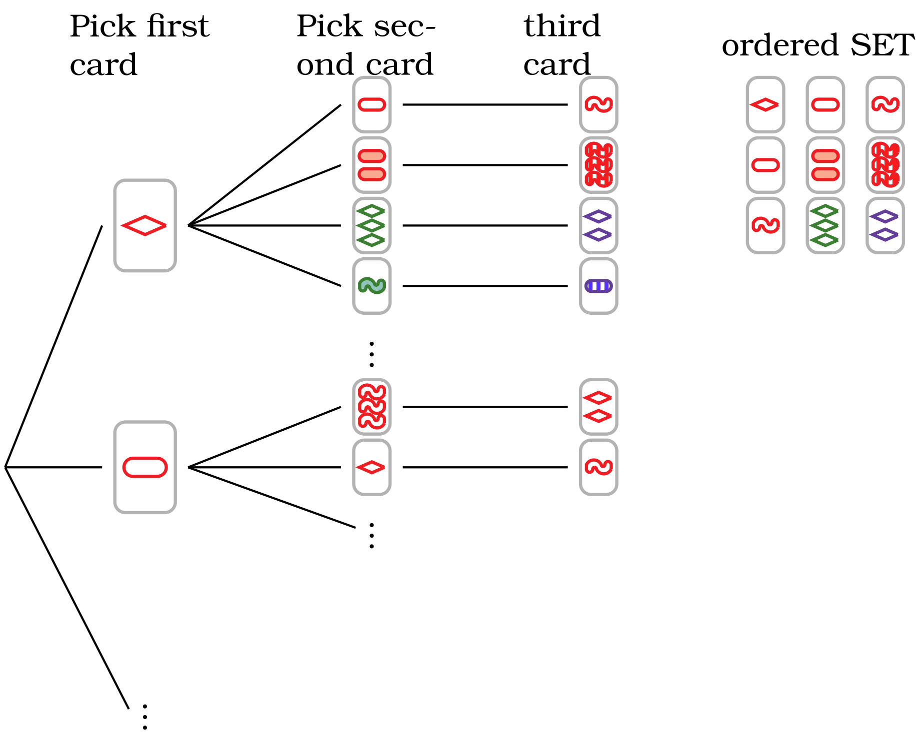
3. If you draw three cards at random, what is the probability that they form a SET?

Since any two cards ______________ a Set, if you draw two cards, then ______________ of the remaining ____ cards will join with the first two cards to make a Set. The probability of such a drawing ________ .

4. How many different ordered SETS are there?

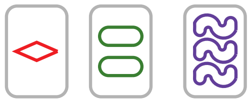
(you can use the blanks to organise your thoughts)

We can use a ______________ to count all **ordered** SETS.



To make a Set, we need to pick 3 cards. There are ________ choices for the first card, ________ choices for the second card, and ________ choice for the third card.

The number of **ordered SET** is ______________

5. Use a tree diagram to find out in how many ways we can order the SET  .

6. How many different (non ordered) SETS are there?

(you can use the blanks to organise your thoughts)

Since the order of cards in a Set doesn't matter, we should divide by ________ .

Thus there are________________ different Sets.

> After playing a few rounds of Set, we start to notice that there are actually four different kinds of Sets. They can be classified according to how many characteristics the cards share: zero, one, two, or three.

7. a) How many cards share no characteristics with  ?

(you can use the blanks to organise your thoughts)

To make such a card, we have to pick among ________ possible choices for number, ________ possible choices for shape, ________ possible choices for shading, and ________ possible choices for color. There are ________________ possible cards.

b) How many cards share exactly 1 characteristics with  ?

(you can use the blanks to organise your thoughts)

To make a card sharing only a same color, we have to pick among ________ possible choices for number, ________ possible choices for shape and ________ possible choices for shading. This makes a ________________ possible cards. Similarly, we have ________________ cards sharing only a same number, as well as ________________ cards sharing only a same shape, and ________________ cards sharing only a same shading.

The total of cards sharing one same characteristic is ________________.

c) How many cards share exactly 3 characteristics with  ?

(you can use the blanks to organise your thoughts)

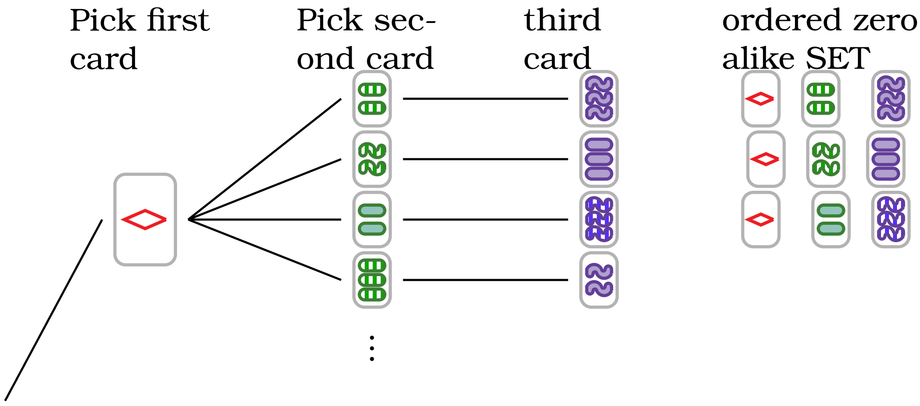
To pick a card sharing same color, number and shape but different shading, we have to pick among ________ possible choices.

The total number of cards sharing 3 characteristics with  is ________________.

d) How many cards are left? How many characteristics do they share with  ?

8. Figure out the percentage of none-alike Sets.

We can use a ______________________________ to count all **ordered** SETS.



The total number of ordered zero-alike sets is _____ $\times$ _____ $\times$ _____ $=$ _____

Since the order of cards in a Set doesn't matter, there are _______________ different none-alike Sets. The percentage among all Sets is _______________ $\approx$ __________%.

9. Figure out the percentage of :

a) one-alike Sets,

b) three-alike Sets among all Sets.

c) two-alike Sets,

Figure 4.3: Hint: The total number of ordered one-alike sets is _____×_____×_____=_____

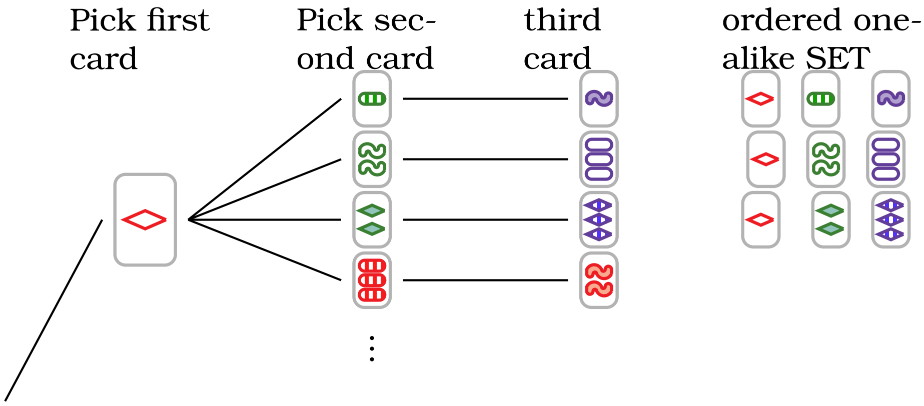


Figure 4.4: Hint: The total number of three-alike sets is _____×_____×_____=_____

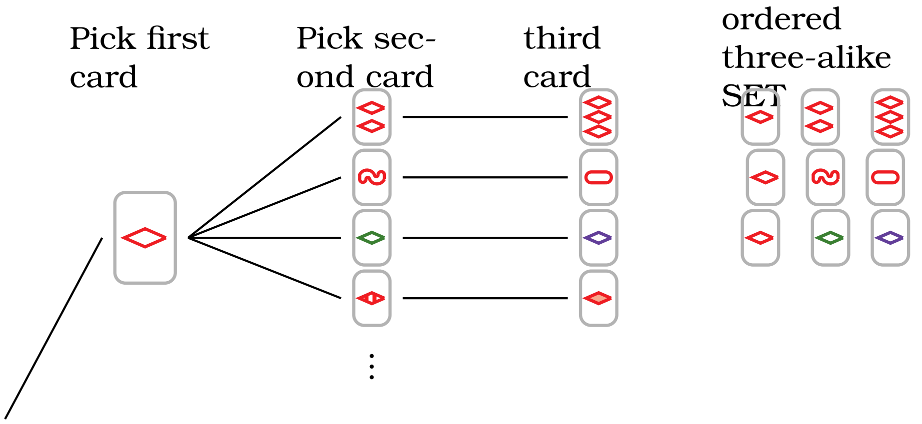
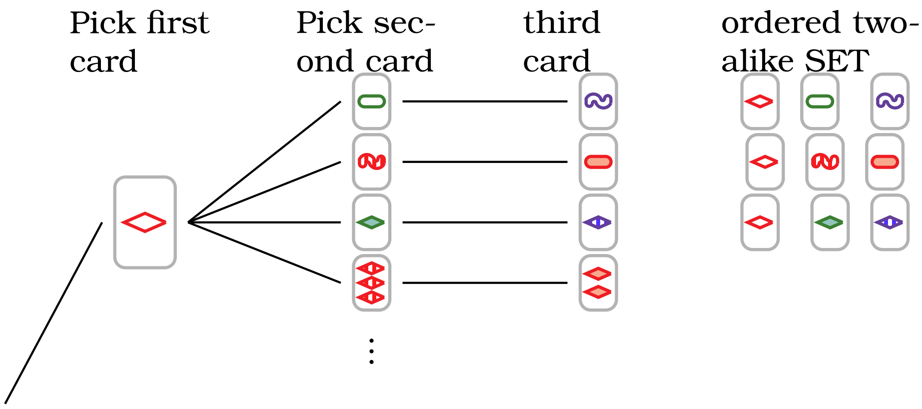


Figure 4.5: Hint: The total number of ordered two-alike sets is _____×_____×_____=_____



Answers

1. How many different cards in a SET deck?

   Since there are three possible values for each of the four characteristics, there are a total of 34 = 81 cards in a Set deck.

2. Show that for every two cards there is a unique third card that will make a set.

   Take the characteristic NUMBER. If the first two cards have the same number, then the third card must have that same number. If the first two cards have different numbers, then the third card must have a number different from the first two and there is only one possibility for that. Similarly for the other three characteristics.

3. If you draw three cards at random, what is the probability that they form a SET?

   Since any two cards uniquely determine a Set, if you draw two cards, then exactly one of the remaining 79 cards will join with the first two cards to make a Set. The probability of drawing it is $1/79$.

4. How many different SETS are there?

   There are 81 choices for the first card, 80 choices for the second card, and one choice for the third card. Since the order of cards in a Set don't matter, we should divide by 3! = 6. Thus there are $81 \times 80/6 = 1080$ different Sets.

5. After playing a few rounds of Set, you will notice that there are different kinds of SETS. What are they? Which kinds do you think are the most and least common? Which seem to be the trickiest to spot?

   There are actually four different kinds of Sets. They can be classified according to how many characteristics the cards share: zero, one, two, or three. The three-alike Sets are easiest to spot. An example of a three-alike Set is "one plain red squiggle," " two plain red squiggles," and "three plain red squiggles." The none-alike or all-different Sets are the most challenging to find. An example of an all-different Set is "one plain red oval," "two striped green squiggles," "three solid purple diamonds."

6. Figure out the percentage of three-alike Sets, two-alike Sets, one-alike Sets, and all-different Sets. If your calculation is correct you should have found that out of the 1080 total possible Sets the number of three-alike Sets is 108, or 10%, the number of two-alike Sets is 324, or 30%, the number of one-alike Sets is 432, or 40%, and the number of all-different Sets is 216 or 20%.

4.7 A mathematical investigation of Dobble!

Let's play Spot it!

1. Play the game a bit at your table. What do you notice about the game? Is it fun? What ages is it appropriate for?

2. How many mathematical questions about the game can you come up with?

DO NOT TURN THE PAGE UNTIL INSTRUCTED

How does Spot it work?

1. How many symbols are on each card?

   There are ....

2. Are there the same number of symbols on each card? Why?

3. How many cards are in the deck? Why? Could there be more? Is there a limit?

   The instruction sheet/to be included/exactly/especially/deck/to exhibit

4. How was the game made?

5. How many times does each symbol appear? Is it the same number for all symbols?

6. How many total symbols are there?

7. Could there be more?

8. Can these questions be answered without putting down some assumptions about the game? What are these assumptions? What are the implicit rules about the game that are in place from the game designers?

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Formal Assumptions and Exploration

Our **Primary Goal** is to **Understand how to construct a Dobble set with a given number of symbols per card**. More precisely, we would like to investigate:

- How many symbols are needed in total?
- How many cards can be made in total?
- How can such a card set be constructed?

We will explore some simpler setups before moving on to larger values of s. We write :

- s for the number of symbols per card
- p for the number of different symbols (the symbol pool).
- c is the number of cards

Let's build a game of Dobble, with s symbols per card. We start with the following assumptions. We might need to add more assumptions to rule out boring solutions.

Assumption 1 **Each card has the same number of symbols on it**

Assumption 2 **Each pair of cards has exactly one matching symbol**.

Assumption 3 **Each card is unique**

Assumption 4 **No symbol appears more than once on a given card**.

Assumption 5 **Each symbol must appear on at least one card** (all symbols are used).

1. Let $s = 2$.

   Check that the following deck satisfies our assumptions. Explain why playing with it would be boring!

	A	B	C	D	E
Card nº 1 has ...	✖	✖	.	.	.
Card nº 2 has ...	✖	.	✖	.	.
Card nº 3 has ...	✖.	.	.	✖	.
Card nº 4 has ...	✖	.	.	.	✖
	.	.	.	.	.
	.	.	.	.	.

Assumption 6 **No symbol is common for all cards**.

Without using new symbols, try adding a card to this deck that doesn't have the symbol A. Remove cards that wouldn't satisfies our assumptions.

Is your deck as big as it possibly could be? What is the total number of symbols used.

How can you represent this deck?

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2. Let $s = 3$. We start by using $p = 7$ symbols.

	A	B	C	D	E	F	G	.	.
Card nº 1 has ...	✖	✖	✖	.	.	.	.	.	.
Card nº 2 has ...	.	.	.	.	.	.	.	.	.
Card nº 3 has ...	.	.	.	.	.	.	.	.	.
Card nº 4 has ...	.	.	.	.	.	.	.	.	.
Card nº 5 has ...	.	.	.	.	.	.	.	.	.
Card nº 6 has ...	.	.	.	.	.	.	.	.	.
Card nº 7 has ...	.	.	.	.	.	.	.	.	.
Card nº 8 has ...	.	.	.	.	.	.	.	.	.
Card nº 9 has ...	.	.	.	.	.	.	.	.	.
Card nº 10 has ...	.	.	.	.	.	.	.	.	.
Card nº 11 has ...	.	.	.	.	.	.	.	.	.

Cards 1 to 3 are $A-$list cards. They all share symbol A.

Add two more $A-$list cards.

$B-$list cards are all cards starting with symbol B.

Create two unique $B-$list cards, and two unique $C-$list cards.

Can we create a E-list card ? (first symbol is E)? Explain.

Why are all possible cards either A-list, B-list or C-list ?

Explain why we can't create a fourth unique $A-$ list card using the available 7 symbols.

.... (to share/to chose)

Explain why we can't create a fourth unique $A-$list card by adding more symbols.

Since there is nothing special about the symbol A, the maximum number of cards that match on **any given symbol** is ....

Since all symbols appear once in the original A-list, they can only appear twice more is the following B-list, C-list... The maximum number of total cards is $c = \ldots + \ldots \times$ ....

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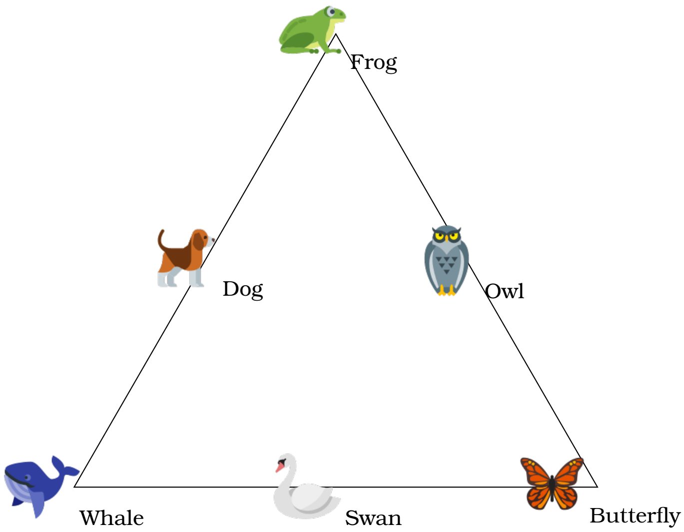
Geometry for $s = 3$ We arranged $p = 6$ symbols in a triangle. Each node represent a symbol :

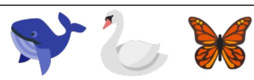
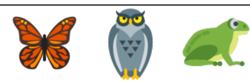
- each card is represented by a line joining 3 nodes : three symbols on a card
- there are no two lines that go through the same two nodes.

a) Why is the card  not admissible?

b) Place symbol  at the center of the triangle and find 3 more cards.

c) **Challenge** : Can you draw a "line" that represents the seventh remaining card?

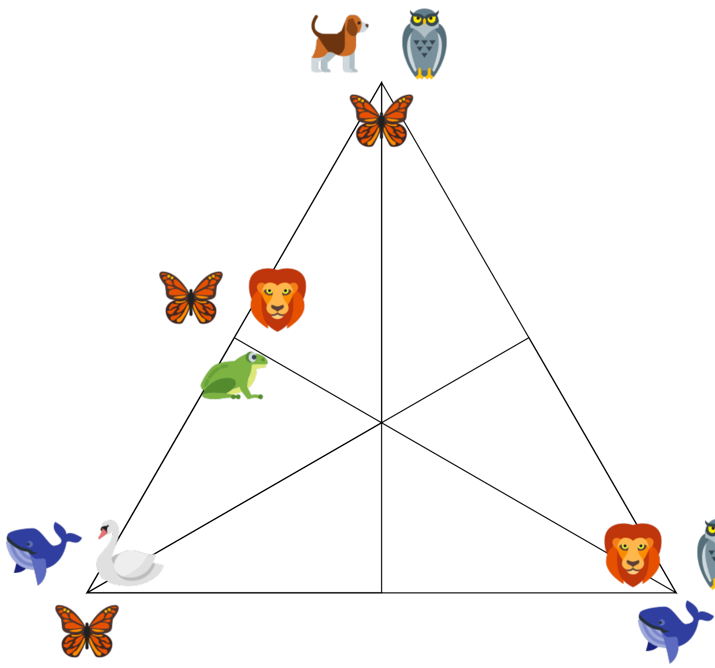


Card nº 1	
Card nº 2	
Card nº 3	
Card nº 4	
Card nº 5	
Card nº 6	
Card nº 7	

The previous figure is called a **Fano plane**.

We can also swap and represent symbols as nodes, and each line joining 3 nodes as a symbol.

- all cards on same line share a single symbol : three different lines go through each card.
- two lines intersect once : no two cards share same two symbols.
- three lines goes through a node : three symbols per card



Find all three missing cards.

3. $s = 4$. Extend previous work to this case. You can fill the following table.

	A	B	C	D	E	F	G	H	I	J	K	L	M
Card nº 1 has ...	✖	✖	✖	✖	.	.	.	.	.	.	.	.	.
Card nº 2 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 3 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 4 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 5 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 6 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 7 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 8 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 9 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 10 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 11 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 12 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 13 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 14 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 15 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.
Card nº 16 has ...	.	.	.	.	.	.	.	.	.	.	.	.	.

Create

- 3 unique $A-$list cards.
- 3 unique $B-$list cards.
- 3 unique $C-$list cards.
- 3 unique $D-$list cards.

Explain using a reductio ad absurdum argument why we can't create a fifth unique $A-$ list card :

Let us suppose that Card 14 is ....

Card nº 5 ____________ an A-list card.

Card nº 5 shares .... with each of the A-lists cards.

Card 14 must have symbol A and ....

Card 14 has a double-match with another card from the A-list. ....

This contradicts our assumption ....

The maximum number of A-list cards is 4.

Since there is nothing special about symbol A, the maximum number of cards that share any given symbol is ....

The maximum pool of unique cards is $c =$ ...... $+$ ...... $\times$ ....

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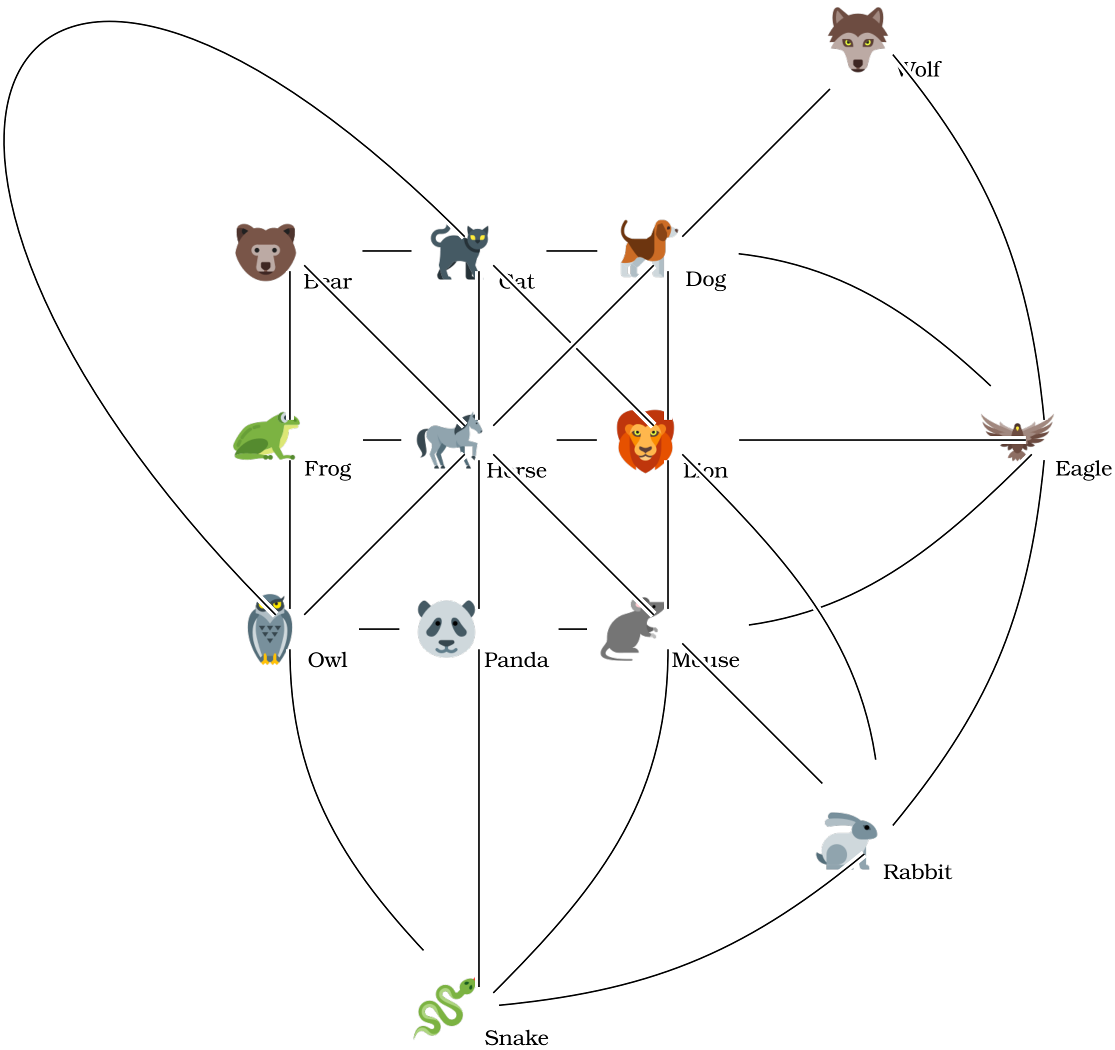
Geometry for $s = 4$ We arranged 13 symbols in a graph. Each node represent a symbol :

- each card is represented by a line joining 4 nodes : four symbols per card
- there are no two lines that go through the same two nodes.
- each node belongs to at most four different "lines".

Challenge How many lines are missing? Can you draw them?



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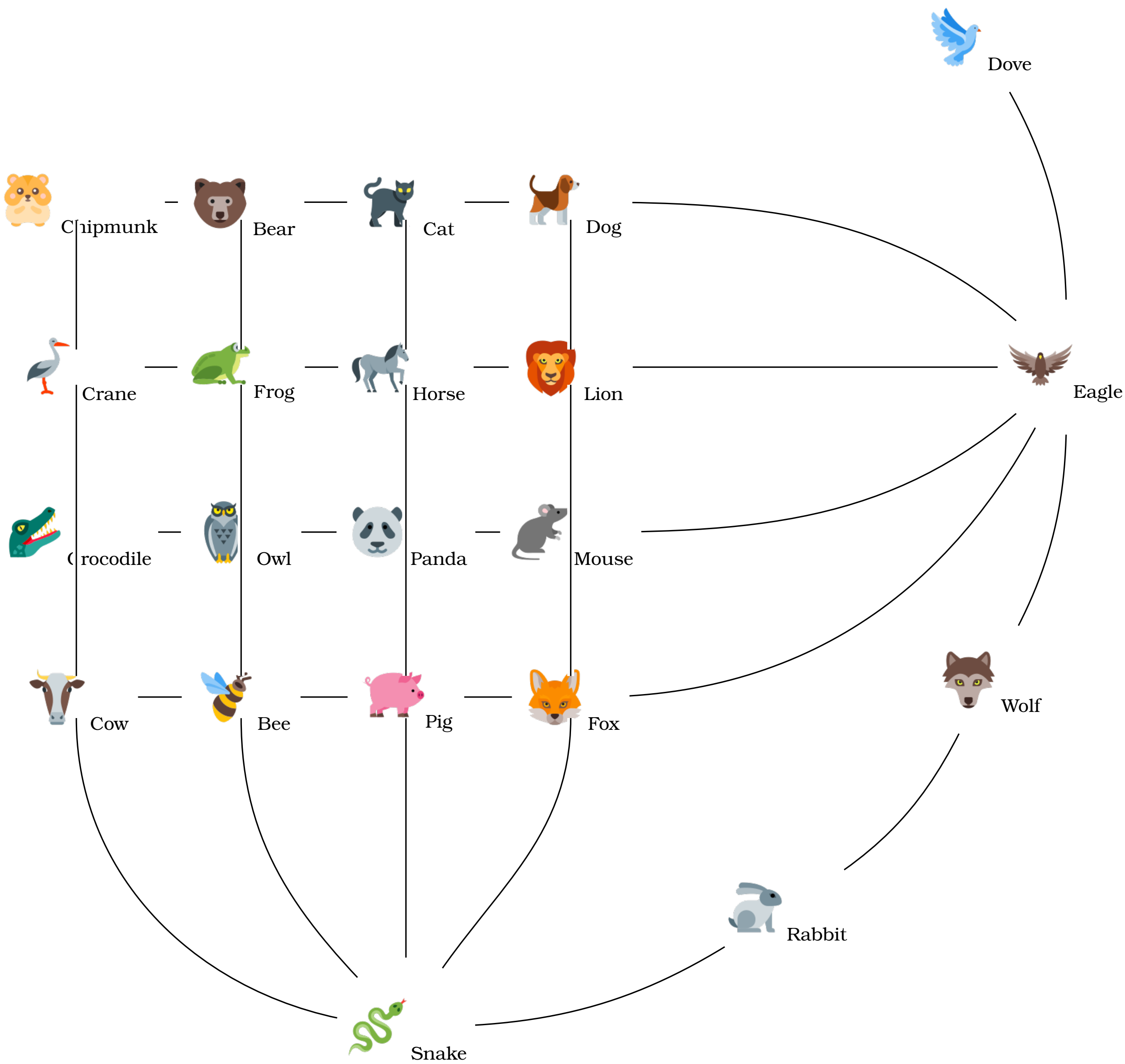
Geometric attempt for $s = 5$ We arranged 16 symbols on a 4×4 grid. We added 4 more cards at infinity. Each node represent a symbol.



Challenge Complete the following cards :

Bear - - - - Rabbit

Bear - - - - Dove

Can you create a fifth card with Bear ?

Draw all lines representing cards with the Dove symbol.

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Case of $s = 5$ symbol per card, our method didn't provide an optimal solution to produce a projective plane. From now on, we take s such that $s - 1$ is a **prime number** :

- $s = 6$ is the junior deck of the Dobble.
- $s = 8$ is the regular deck of the Dobble.

How does Dobble (Spot It) work? by Matt Parker

https://youtu.be/VTDKqW_GLkw?t=216 (Start at 3'30")

1. How did Matt Parker arrange the **regular** Dobble deck ?
2. In his representation what is a node? what does a line represent?
3. How many lines go through each node ? Describe them.
4. What is the total number of symbols used?
5. What is the total number of possible cards?

Challenge

Find the missing card in the $s = 6$ junior Dobble deck. You can start by drawing a 5×5 grid on the brown paper, and pick any 4 different cards on the top right corner.

Projective plane

- Two distinct points are contained in a unique line.
- In a projective plane, there are no parallel lines. All lines intersect in a unique point.
- A projective plane of order n has $n^2 + n + 1$ points and lines, with $n + 1$ points on each line, and $n + 1$ lines on each point.

Generalizing formulas

The **pigeonhole principle** states that if n items are put into m containers, with $n > m$, then at least one container must contain more than one item.

- s the number of symbols per card
- p the number of different symbols
- c is the number of cards

We rank symbols such that our first card has all first s symbols.

A $n-$ list are all cards that start with the symbol n.

For example, $1-$ list are cards that start with symbol 1.

$2-$ list are cards that start with symbol 2. And so on.

All cards must be in one of $1-$list up to $s-$list.

Statement 1 The maximum number of 1-list unique cards is equal to s.

Proof. The proof is by *reductio ad absurdum*

Let us suppose there are $s + 1$ unique 1-list cards.

By assumption ......, there is a card with s symbols that isn't an 1-list.

Such a card has .... (matching symbol)

By the pigeonhole principle at least two cards from 1-list have a **double match** : they match on symbol 1, and on some other symbol.

....

■

Statement 2 The maximum number of unique cards is $s^2 - s + 1$.

Since there is nothing special about the symbol 1, the maximum number of cards that match on any symbol is ....

The 2-symbol appeared once in the 1-list, it can only appear ....more. $(s/s + 1/s - 1)$.

The 3-symbol appeared once in the A-list, it can appears ....more. $(s/s + 1/s - 1)$.

And so on.

The maximum number of unique cards is $c = \ldots\ldots + \ldots\ldots\ldots \times \ldots\ldots\ldots$ ....

$s = 6$ is the junior deck of Dobble. The maximum number of unique cards that we can create is $c =$ ....

$s = 8$ is the regular deck of Dobble. The maximum number of unique cards that we can create is $c =$ ....

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Statement 3 The maximum number of cards is less than the size of the pool of symbols : $c \leqslant p$.

Proof.

Total number of symbols on all cards is ....

Each symbol appears ....(less than/more than)

.... ■

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
nº 1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 7	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 9	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 10	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 12	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 13	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 14	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 15	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 17	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 18	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 19	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 20	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 21	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
nº 1	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.
nº 2	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.
nº 3	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.
nº 4	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.
nº 5	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X
nº 6	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.
nº 7	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.	X
nº 8	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.	.
nº 9	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.	.
nº 10	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.	.
nº 11	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.	.
nº 12	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.	.
nº 13	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.	.
nº 14	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.	.
nº 15	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.	.
nº 16	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X	.
nº 17	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.	X
nº 18	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.	.
nº 19	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X	.
nº 20	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X	X
nº 21	X	.	.	X	.	.	.	.	.	.	.	.	.	X	.	X	.	.	.	.	X

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
nº 1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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nº 4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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nº 12	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 13	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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nº 19	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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nº 22	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 23	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 24	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 25	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 26	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 27	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 28	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 29	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 30	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
nº 31	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

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